Lecture 7:

Wavefront Sensing

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Outline of lecture

- General discussion: Types of wavefront sensors

- Three types in more detail:
  - Shack-Hartmann wavefront sensors
  - Curvature sensing
  - Pyramid sensing

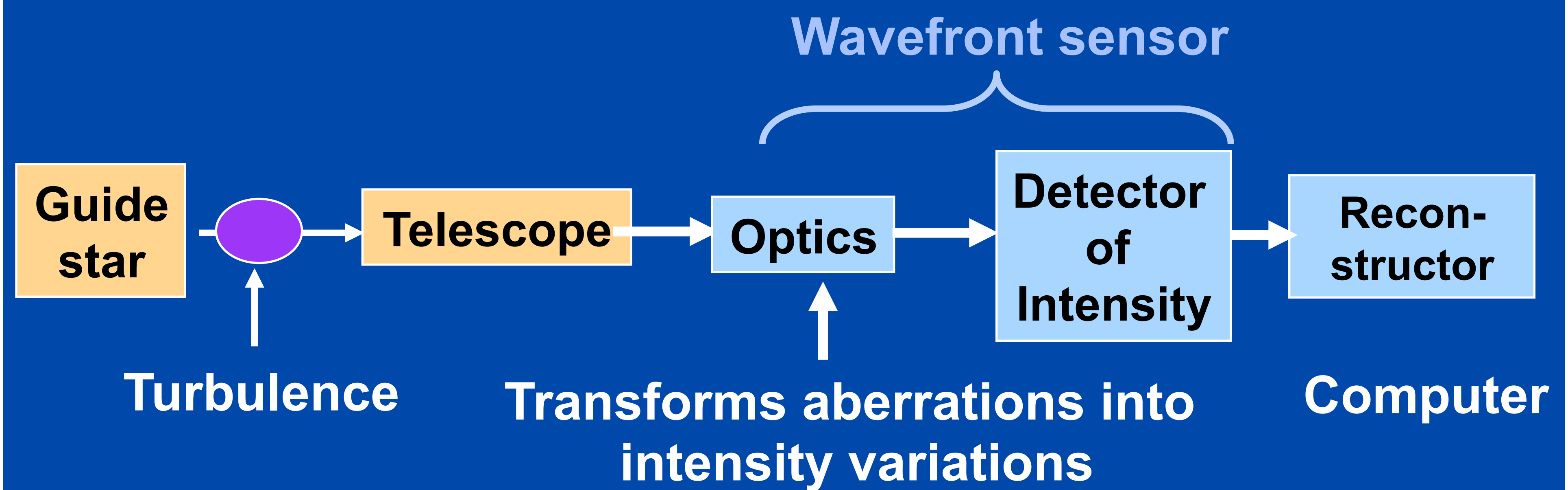
At longer wavelengths, one can measure phase directly

- FM radios, radar, radio interferometers like the VLA, ALMA
- All work on a narrow-band signal that gets mixed with a very precise "intermediate frequency" from a local oscillator
- Very hard to do this at visible and near-infrared wavelengths
  - Could use a laser as the intermediate frequency, but would need tiny bandwidth of visible or IR light

Thanks to Laird Close's lectures for making this point

At visible and near-IR wavelengths, measure phase via intensity variations

- Difference between various wavefront sensor schemes is the way in which phase differences are turned into intensity differences
- General box diagram:



How to use intensity to measure phase?

- Irradiance transport equation: *A* is complex field amplitude, z is propagation direction. (Teague, 1982, JOSA 72, 1199)

$$\text{Let } A(x,y,z) = [I(x,y,z)]^{1/2} \exp[ik\phi(x,y,z)]$$

- Follow *I (x,y,z)* as it propagates along the z axis (paraxial ray approximation: small angle w.r.t. z)

$$\frac{\partial I}{\partial z} = -\nabla I \bullet \nabla\phi - I\nabla^2\phi$$

Wavefront curvature: Curvature Sensors

Wavefront tilt: Hartmann sensors

Types of wavefront sensors

- **"Direct" in pupil plane:** split pupil up into subapertures in some way, then use intensity in each subaperture to deduce phase of wavefront. Sub-categories:
  - Slope sensing: Shack-Hartmann, lateral shear interferometer, pyramid sensing
  - Curvature sensing
- **"Indirect" in focal plane:** wavefront properties are deduced from whole-aperture intensity measurements made at or near the focal plane. Iterative methods – calculations take longer to do.
  - Image sharpening, multi-dither
  - Phase diversity, phase retrieval, Gerchberg-Saxton (these are used, for example, in JWST)

How to reconstruct wavefront from measurements of local "tilt"

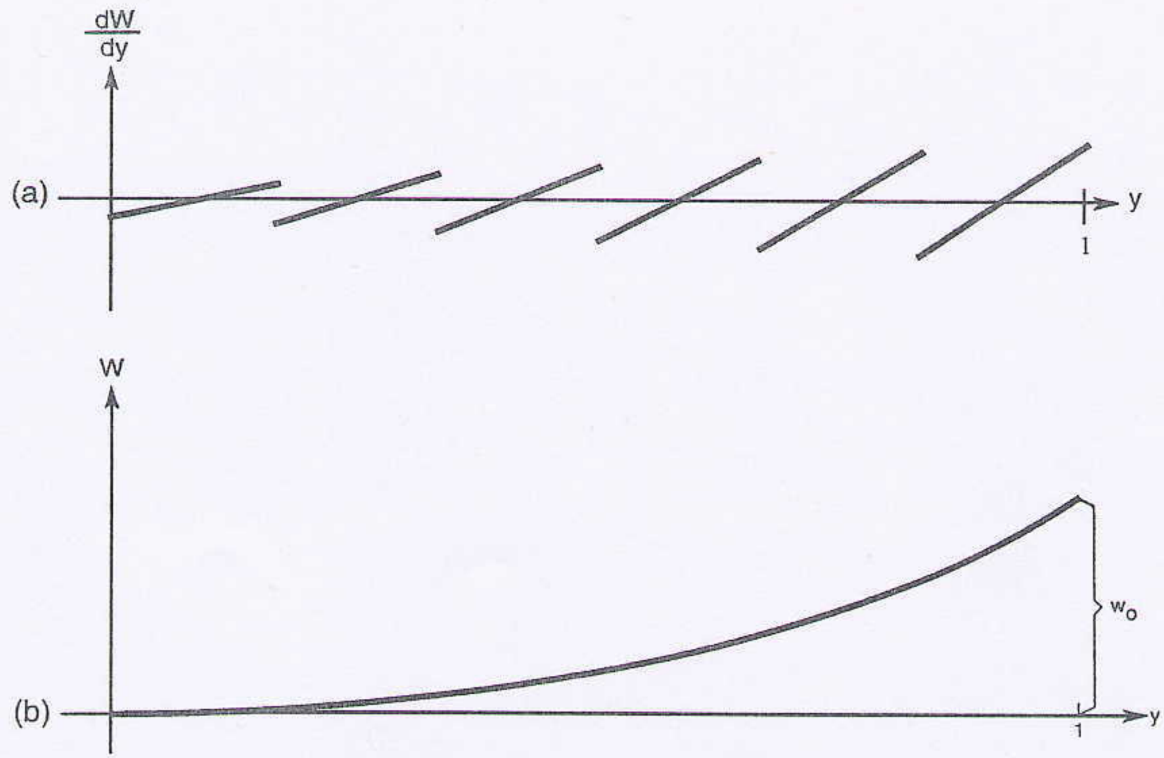
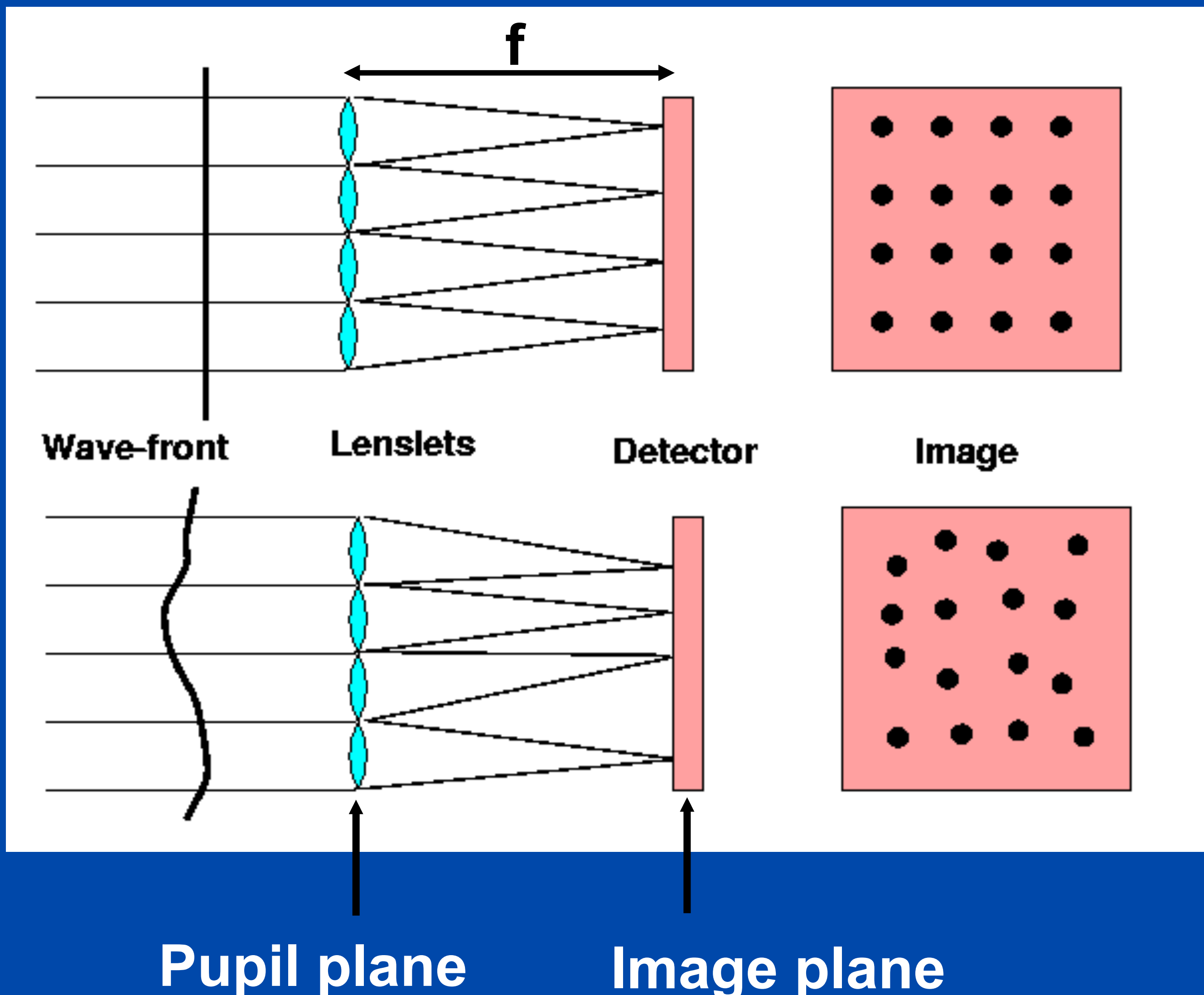


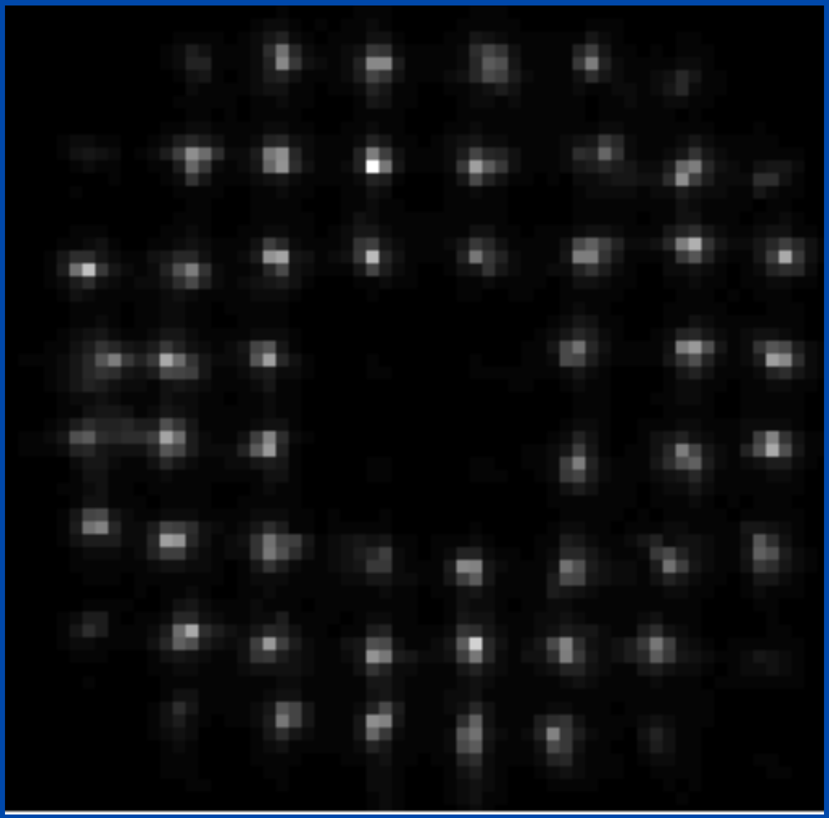
Figure 7 (a) Local tilt as a function of sampling location in pupil; (b) reconstructed wavefront estimate.

Shack-Hartmann wavefront sensor concept - measure subaperture tilts



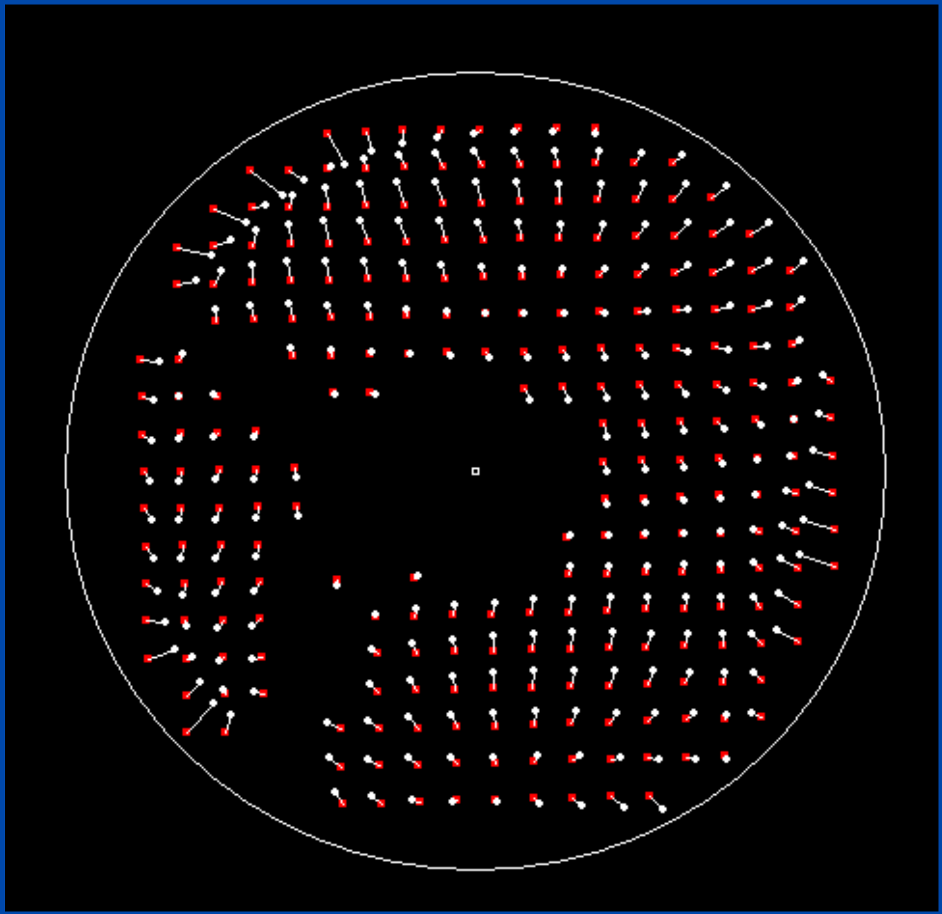
Credit:
A. Tokovinin

Example: Shack-Hartmann Wavefront Signals



Credit: Cyril Cavadore

Displacement of centroids



Credit: Cyril Cavador

- Definition of centroid

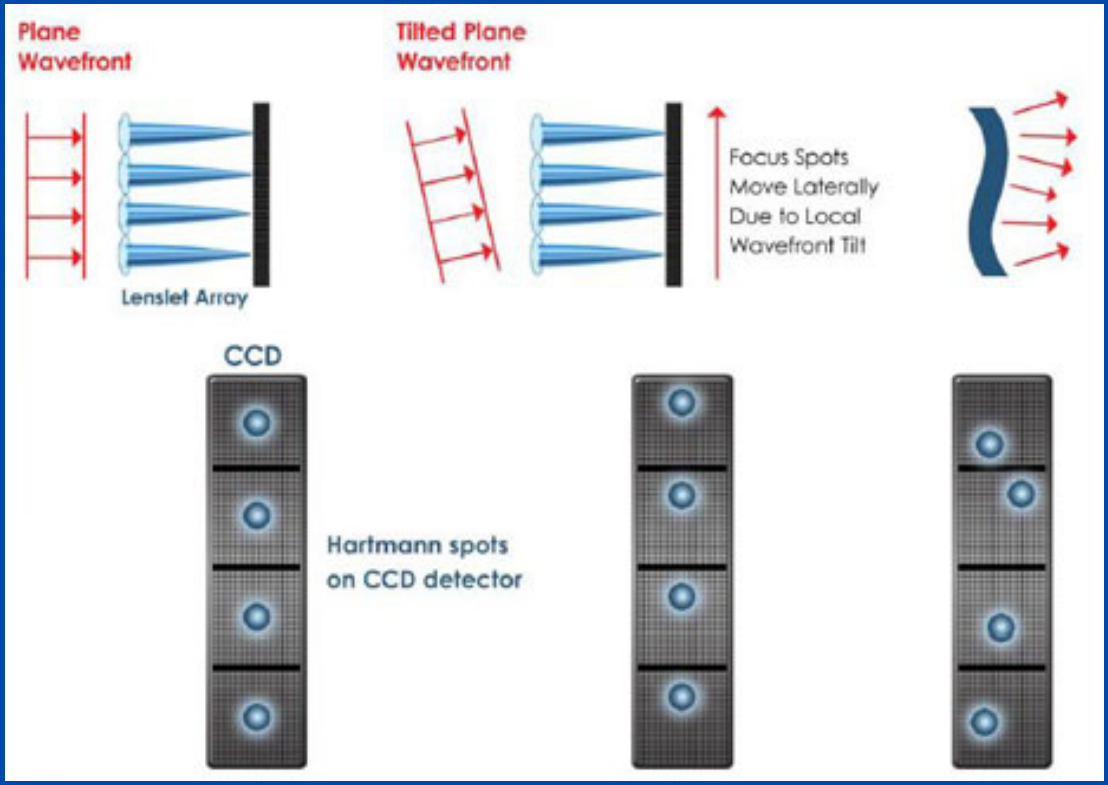
$$\bar{x} \equiv \frac{\int\int I(x,y)\ x\ dxdy}{\int\int I(x,y) dxdy}$$

$$\bar{y} \equiv \frac{\int\int I(x,y)\ y\ dxdy}{\int\int I(x,y) dxdy}$$

- Centroid is intensity weighted

← Each arrow represents an offset proportional to its length

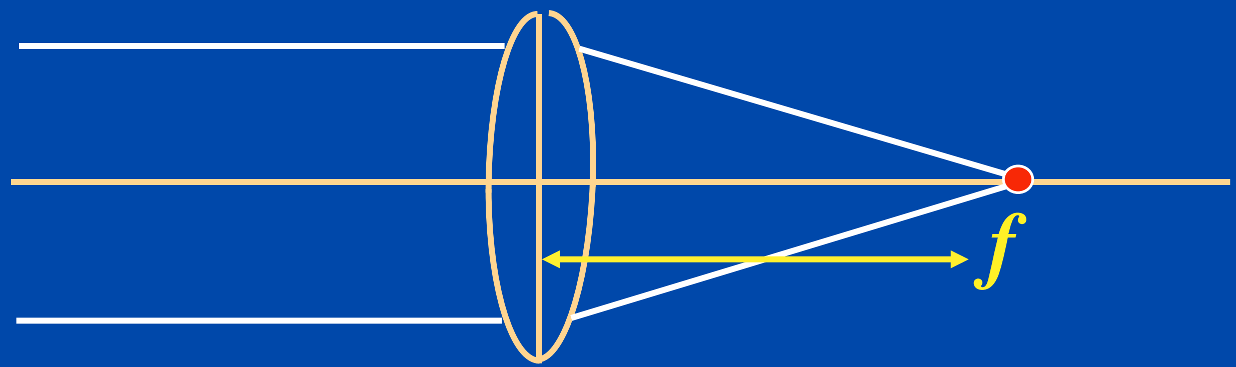
Notional Shack-Hartmann Sensor spots



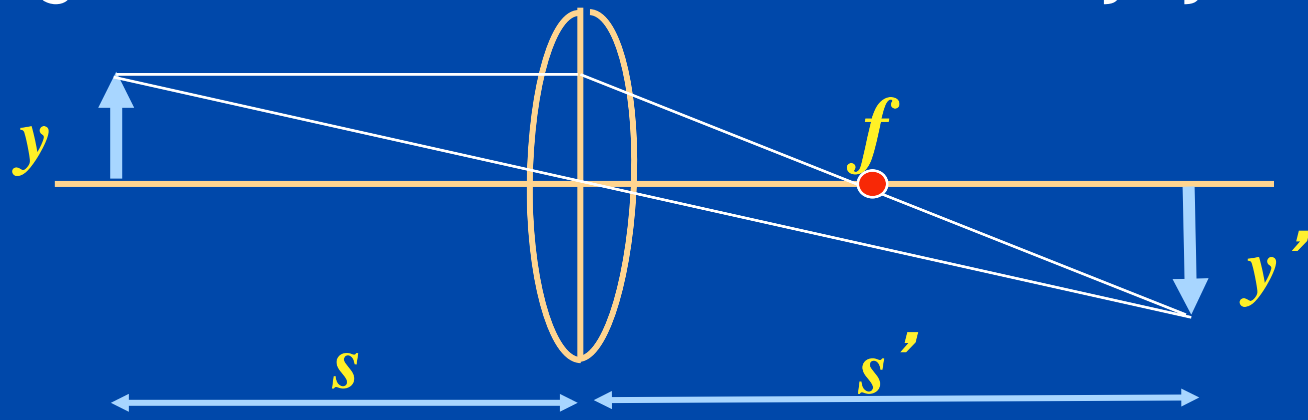
Credit: Boston Micromachines

Reminder of some optics definitions: focal length and magnification

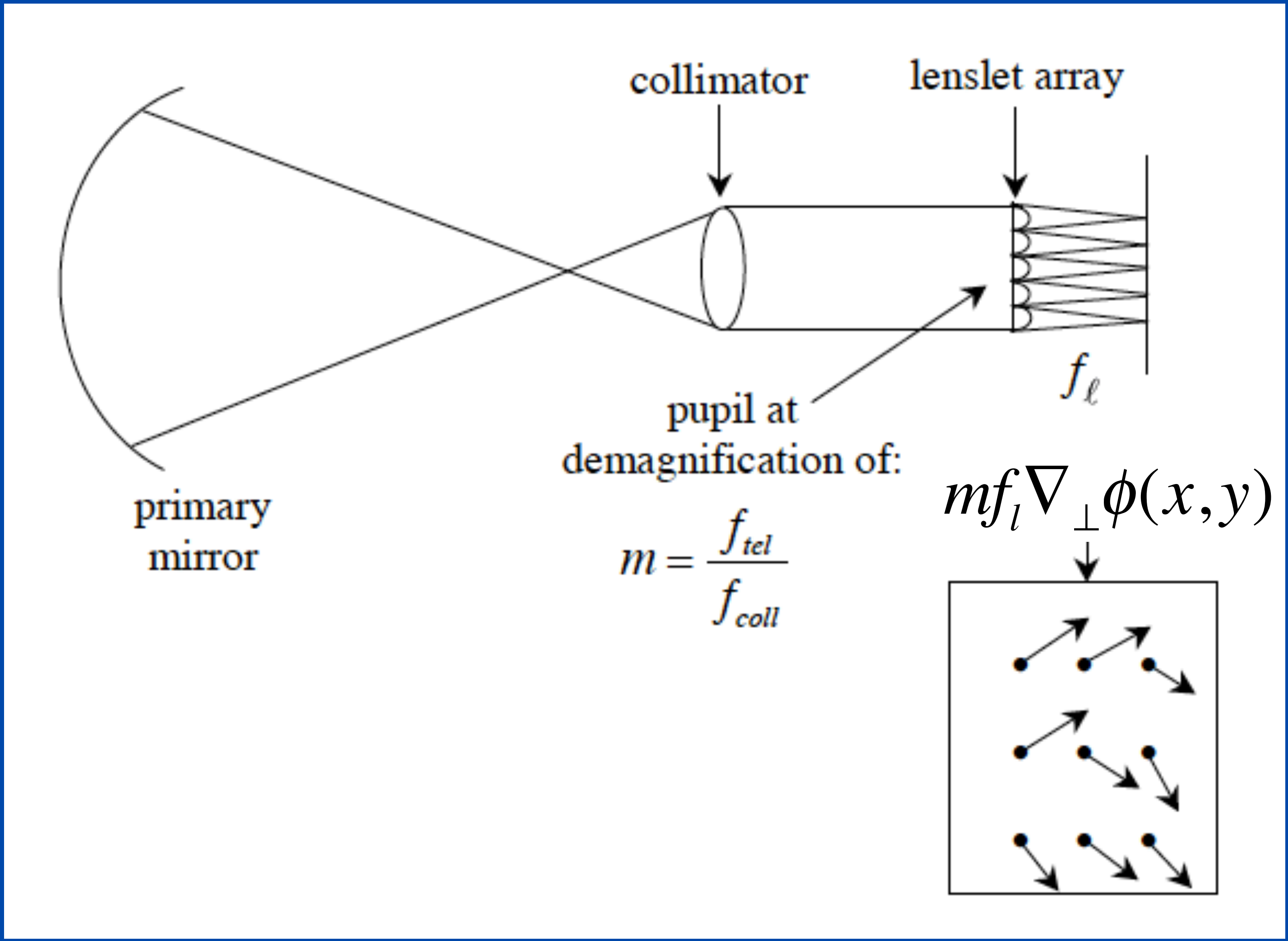
- Focal length f of a lens or mirror



- Magnification $M = y'/y = -s'/s$



Displacement of Hartmann Spots



Quantitative description of Shack-Hartmann operation

- Relation between displacement of Hartmann spots and slope of wavefront:

$$\Delta\vec{x} \propto \nabla_{\perp}\phi(x,y)$$

$$k\Delta\vec{x} = M f \nabla_{\perp}\phi(x,y)$$

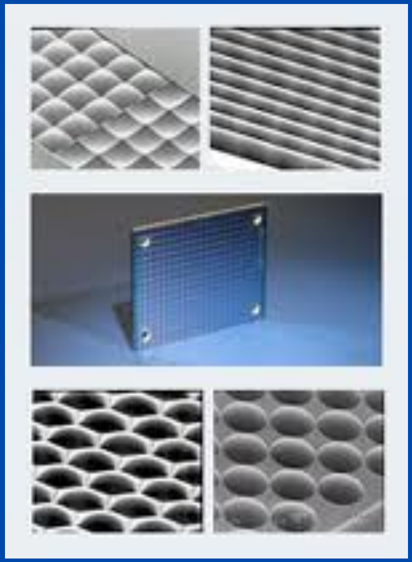
where $k = 2\pi / \lambda$, Δx is the lateral displacement of a subaperture image, M is the (de)magnification of the system, f is the focal length of the lenslets in front of the Shack-Hartmann sensor

Example: Keck adaptive optics system

- Telescope diameter D = 10 m,
  M = 2800 ⇒ size of whole lenslet array
  = 10/2800 m = 3.57 x 10^{-3} m = 3.57mm

- Lenslet array is approx. 18 x 18 lenslets ⇒ each lenslet is ~ 200 microns in diameter

✓ Sanity check: size of subaperture on telescope mirror = lenslet diameter x magnification = 200 microns x 2800 = 56 cm ~ r_0 for wavelength λ between 1 and 2 microns



Some examples of micro-lenslet arrays

Keck AO example, continued

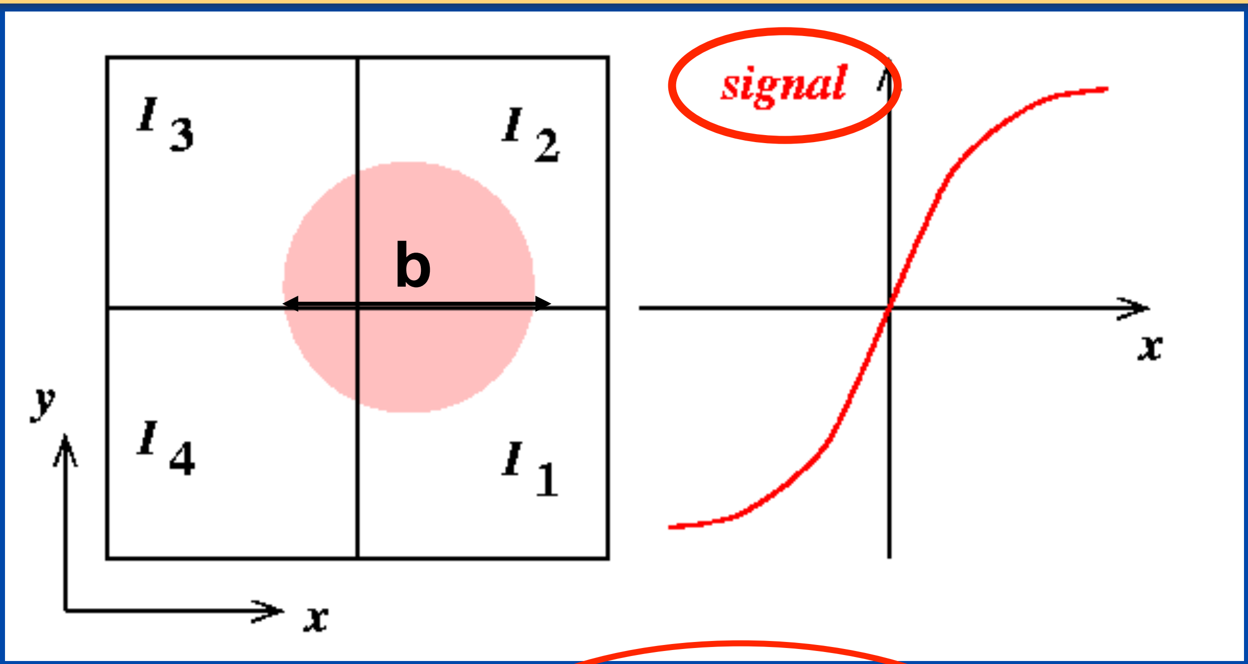
- Now look at scale of pixels on CCD detector:
  - Lenslet array size (200 microns) is larger than size of the CCD detector, so must put a focal reducer lens between the lenslets and the CCD: scale factor 3.15
- Each subaperture is then mapped to a size of 200 microns ÷ 3.15 = 63 microns on the CCD detector
- Choose to make this correspond to 3 CCD pixels (two to measure spot position, one for "guard pixel" to keep light from spilling over between adjacent subapertures)
  - So each pixel is 63/3 = 21 microns across.
- Now calculate angular displacement corresponding to one pixel, using

$$k\Delta\vec{x} = M\ f\ \nabla_{\perp}\phi(x,y)$$

Keck AO example, concluded

- Angle corresponding to one pixel = $\Delta z/\Delta x$ where the phase difference $\Delta\varphi = k\,\Delta z$.
- $\Delta z\ /\ \Delta x$ = (pixel size x 3.15) ÷ (2800 x 200 x 10)
- Pixel size is 21 microns.
- $\Delta z\ /\ \Delta x$ = (21 x 3.15) ÷ (2800 x 2000) = 11.8 microradians
- **Now use factoid: 1 arc sec = 4.8 microradians**
- $\boldsymbol{\Delta z\ /\ \Delta x}$ = 2.4 arc seconds.
- So when a subaperture has 2.4 arc seconds of slope across it, the corresponding spot on the CCD moves sideways by 1 pixel.

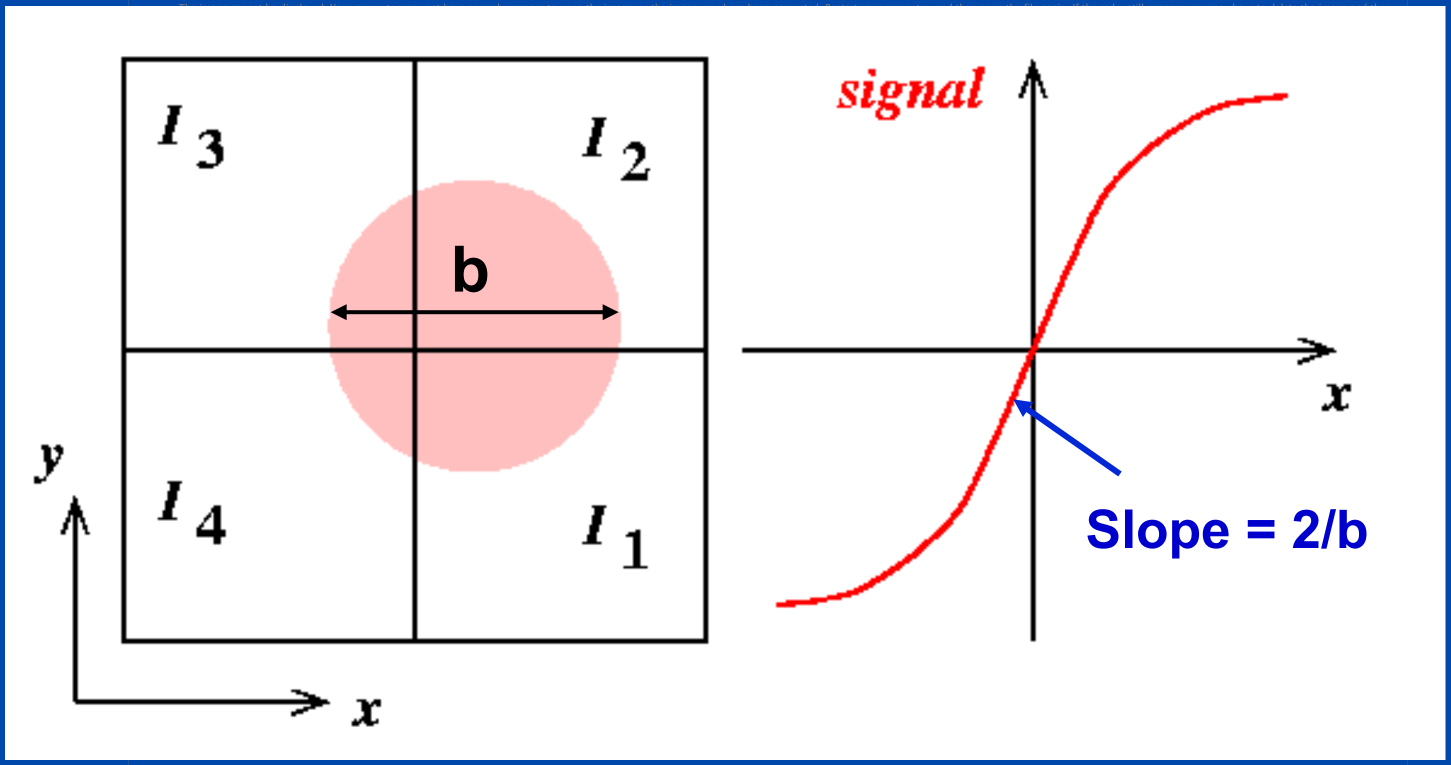
How to measure distance a spot has moved on CCD? "Quad cell formula"



$$\delta_x \cong \frac{b}{2}\left[\frac{(I_2+I_1)-(I_3+I_4)}{(I_1+I_2+I_3+I_4)}\right]$$

$$\delta_y \cong \frac{b}{2}\left[\frac{(I_3+I_2)-(I_4+I_1)}{(I_1+I_2+I_3+I_4)}\right]$$

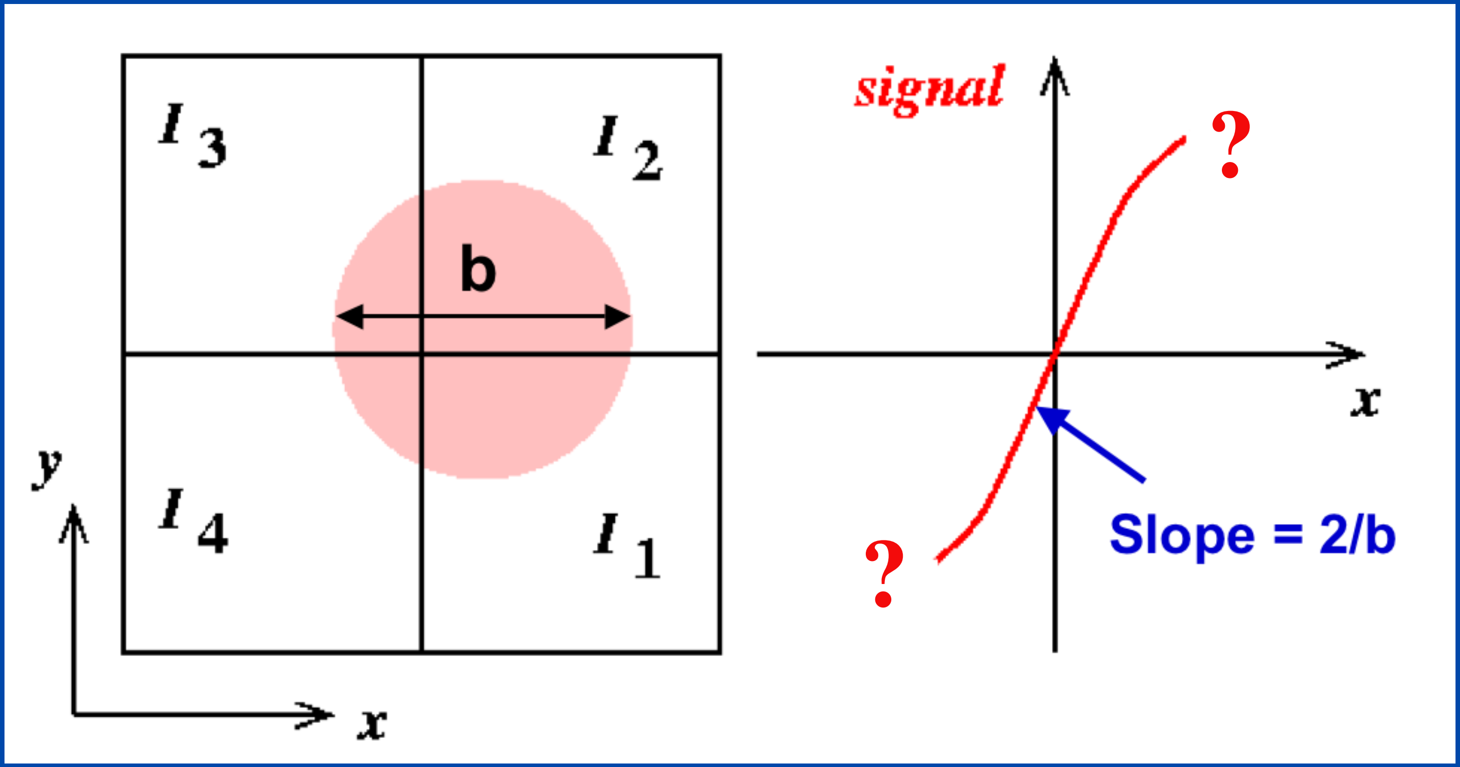
Disadvantage: "gain" depends on spot size b which can vary during the night



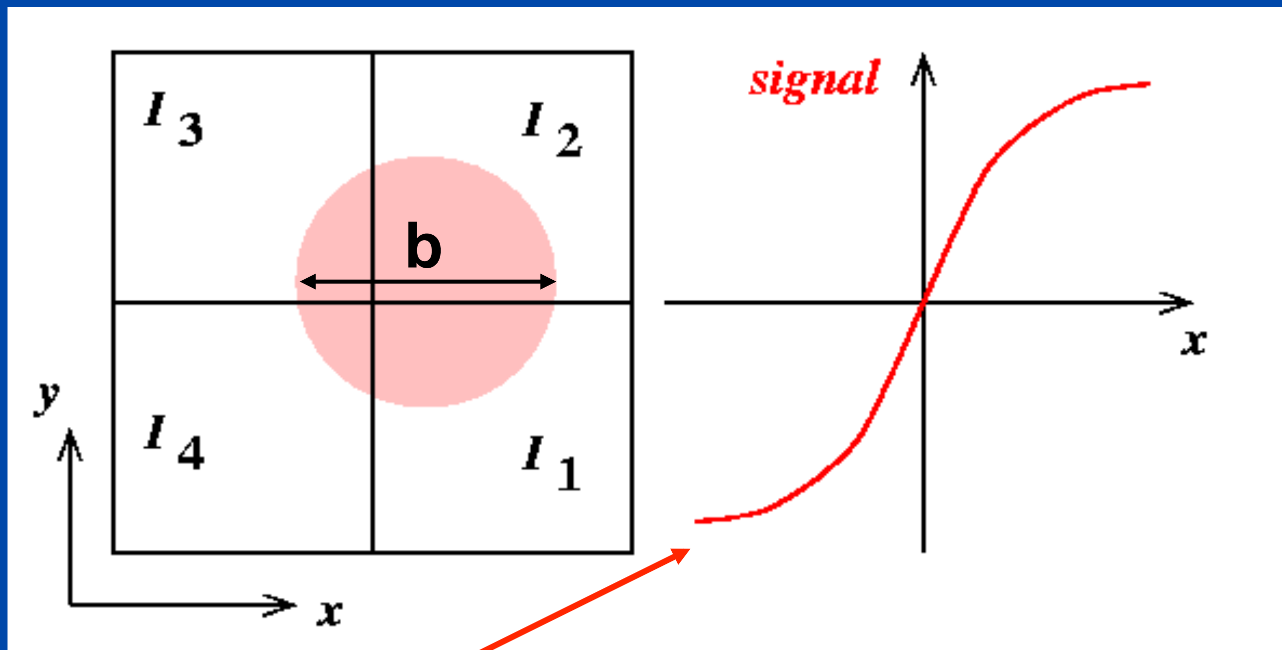
$$\delta_{x,y} = \frac{b}{2} \frac{(difference\ \ of\ I\text{'}s)}{(sum\ of\ I\text{'}s)}$$

Question

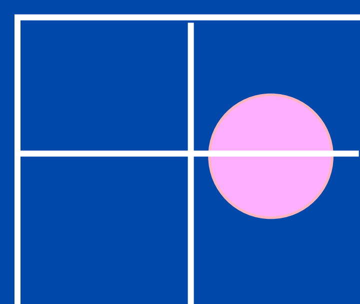
- What might happen if the displacement of the spot is > radius of spot? Why?



Signal becomes nonlinear and saturates for large angular deviations



"Rollover" corresponds to spot being entirely outside of 2 quadrants



Measurement error from Shack-Hartmann sensing

- Measurement error depends on size of spot as seen in a subaperture, θ_b , wavelength λ , subaperture size *d*, and signal-to-noise ratio *SNR*:

$$\sigma_{S-H} = \frac{\pi^2}{2\sqrt{2}} \frac{1}{SNR} \left[\left(\frac{3\mathrm{d}}{2\mathrm{r}_0} \right)^2 + \left(\frac{\vartheta_b d}{\lambda} \right)^2 \right]^{1/2} \text{ rad} \quad \text{for } r_0 \le d$$

$$\sigma_{S-H} \cong \frac{6.3}{SNR} \text{ rad of phase} \quad \text{for } r_0 = d \text{ and } \vartheta_b = \frac{\lambda}{d}$$

(Hardy equation 5.16)

Order of magnitude, for r_0 ~ d

- If we want the wavefront error to be < $\lambda/20$, we need

$$\Delta z \equiv \frac{\sigma}{k} < \frac{\lambda}{20} \quad \text{or} \quad \sigma \cong \frac{6.3}{SNR} < \frac{2\pi}{20} \text{ so that SNR} > 20$$

General expression for signal to noise ratio of a pixelated detector

- S = flux of detected photoelectrons / subap
  n_{pix} = number of detector pixels per subaperture
  R = read noise in electrons per pixel
- The signal to noise ratio in a subaperture for fast CCD cameras is dominated by read noise, and

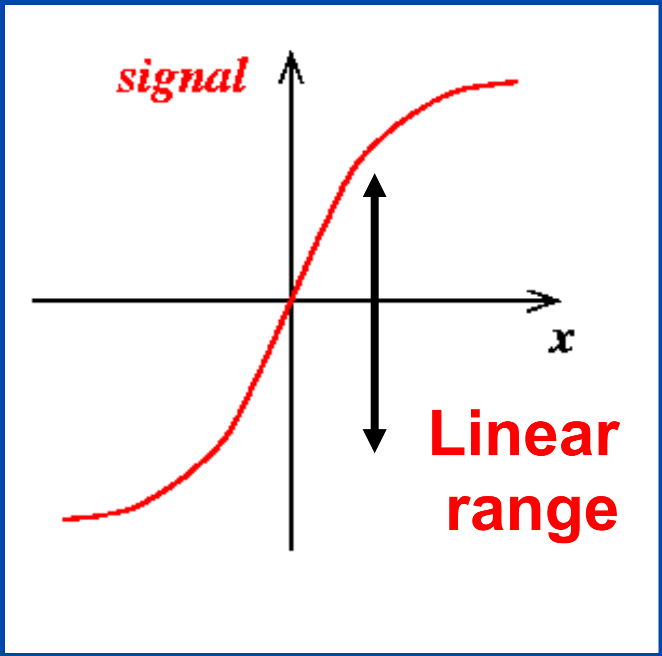
$$SNR \approx \frac{S t_{\text{int}}}{(n_{pix} R^2 / t_{\text{int}})^{1/2}} = \frac{S\sqrt{t_{\text{int}}}}{\sqrt{n_{pix}}\, R}$$

See McLean, "Electronic Imaging in Astronomy", Wiley

We will discuss SNR in much more detail in a later lecture

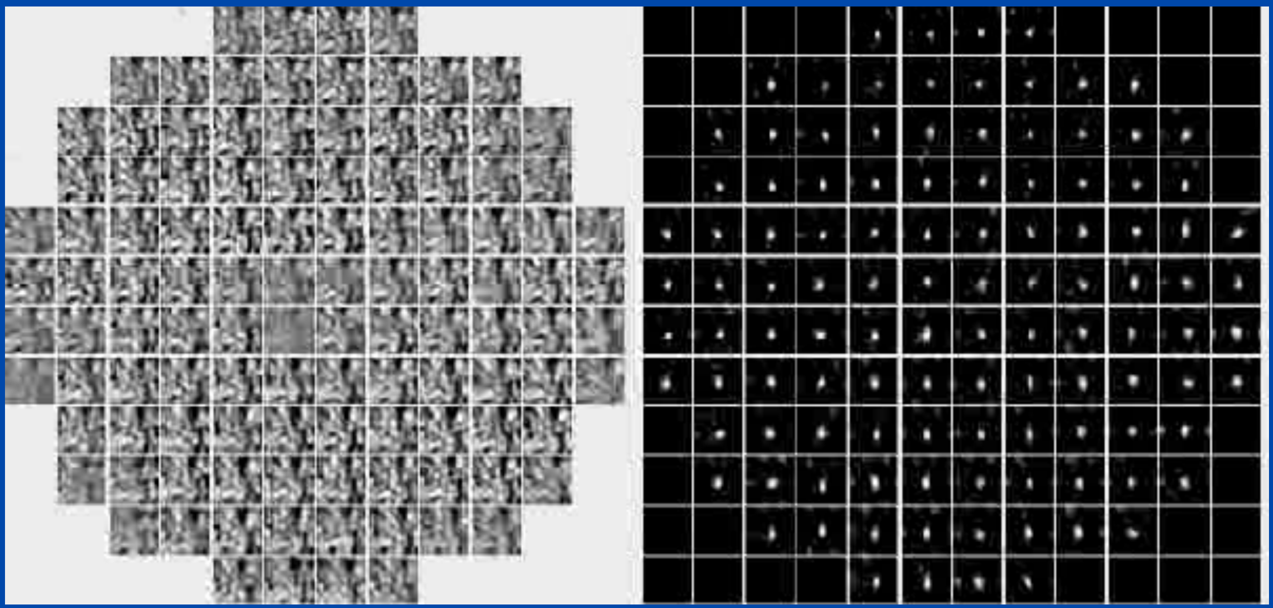
Trade-off between dynamic range and sensitivity of Shack-Hartmann WFS

- If spot is diffraction limited in a subaperture *d*, linear range of quad cell (2x2 pixels) is limited to $\pm \lambda_{ref}/2d$.
- Can increase dynamic range by enlarging the spot (e.g. by defocusing it).
- But uncertainty in calculating centroid $\propto$ width x $N_{ph}^{1/2}$ so centroid calculation will be less accurate.
- Alternative: use more than 2x2 pixels per subaperture. Decreases *SNR* if read noise per pixel is large (spreading given amount of light over more pixels, hence more read noise).



Correlating Shack-Hartmann wavefront sensor uses images in each subaperture

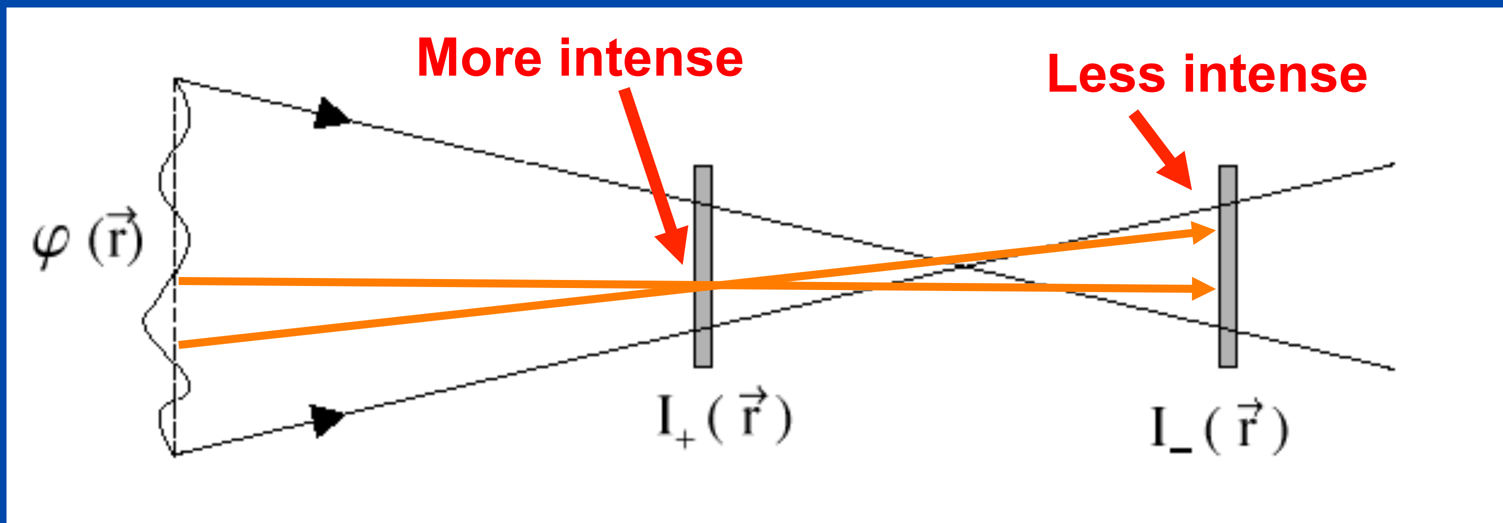
- Solar adaptive optics: Rimmele and Marino
  http://solarphysics.livingreviews.org/Articles/lrsp-2011-2/



- Cross-correlation is used to track low contrast granulation
- Left: Subaperture images, Right: cross-correlation functions

Curvature wavefront sensing

- F. Roddier, Applied Optics, 27, 1223- 1225, 1998



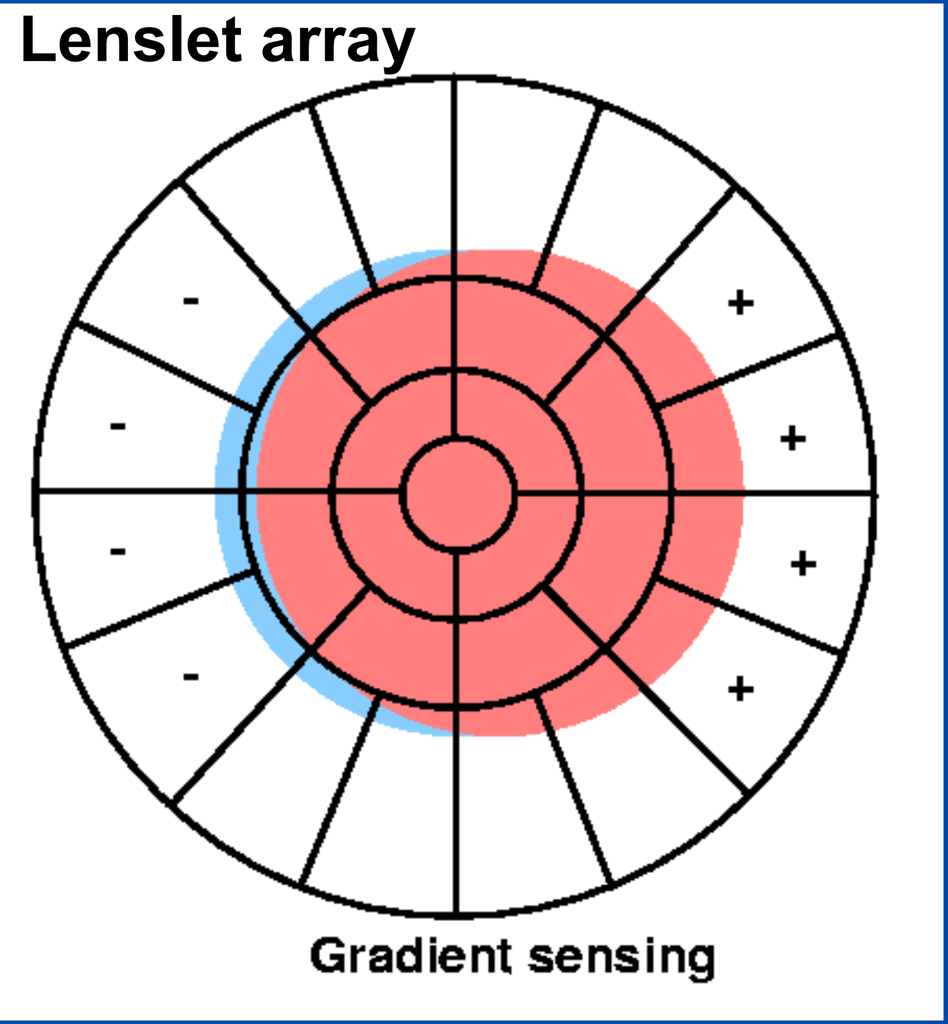
$$\frac{I_+ - I_-}{I_+ + I_-} \propto \nabla^2 \phi - \frac{\partial \phi}{\partial \vec{r}} \vec{\delta}_R$$

Normal derivative at boundary

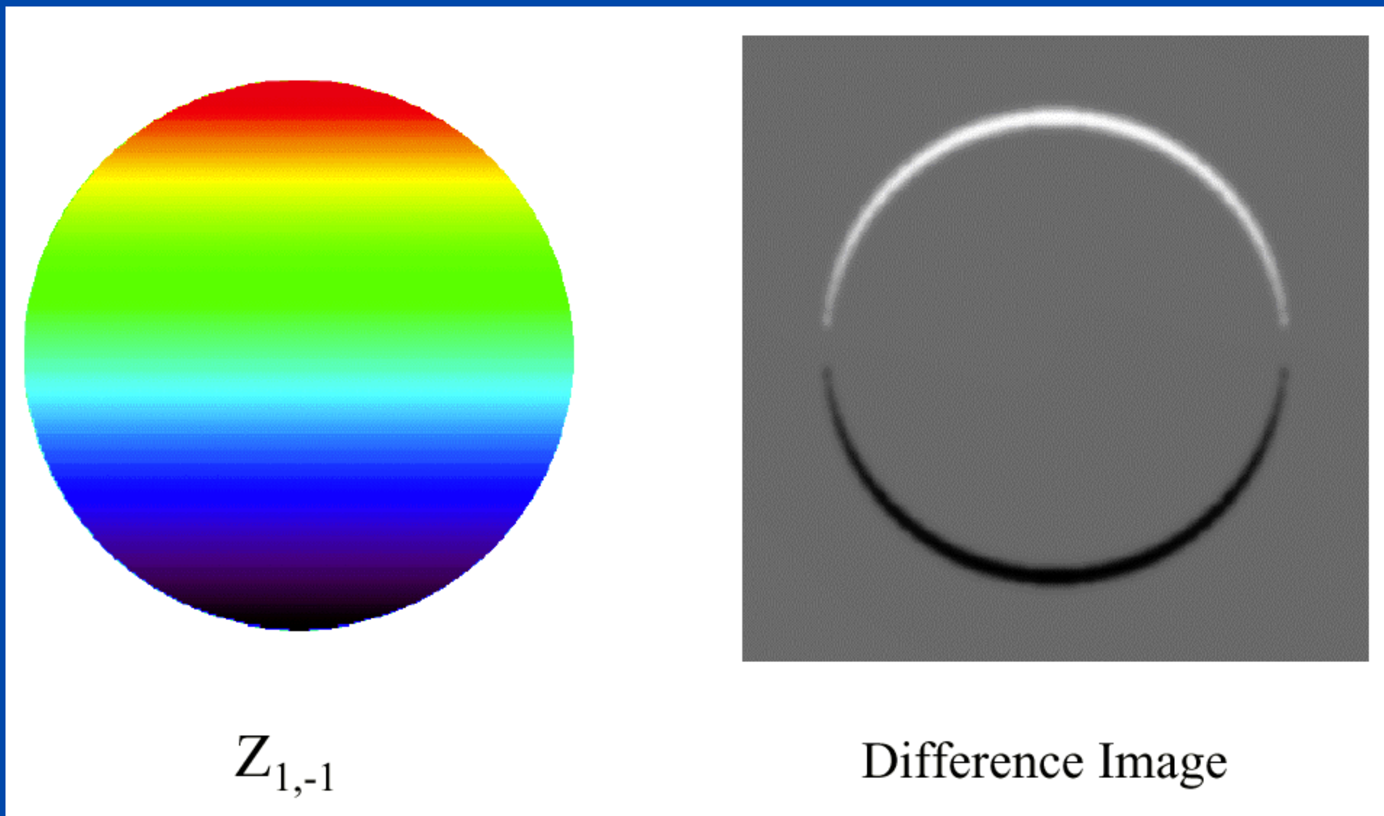
Laplacian (curvature)

Wavefront sensor lenslet shapes are different for edge, middle of pupil

- Example: This is what wavefront tilt (which produces image motion) looks like on a curvature wavefront sensor
  - Constant I on inside
  - Excess I on right edge
  - Deficit on left edge



Simulation of curvature sensor response

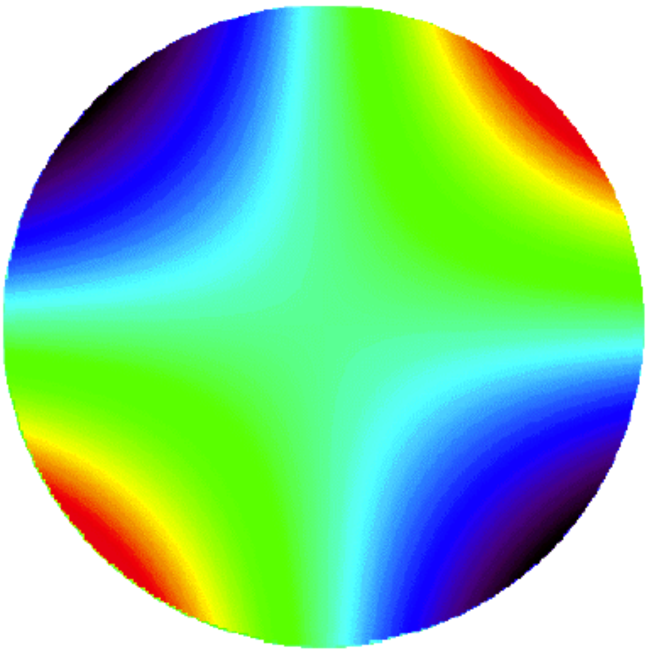


Wavefront: pure tilt

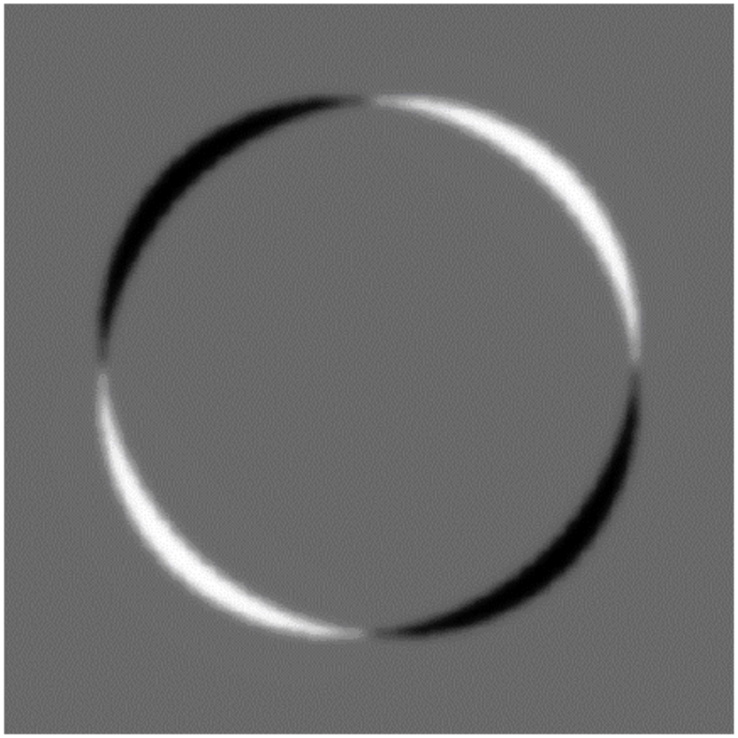
Curvature sensor signal

Credit: G. Chanan

Curvature sensor signal for astigmatism



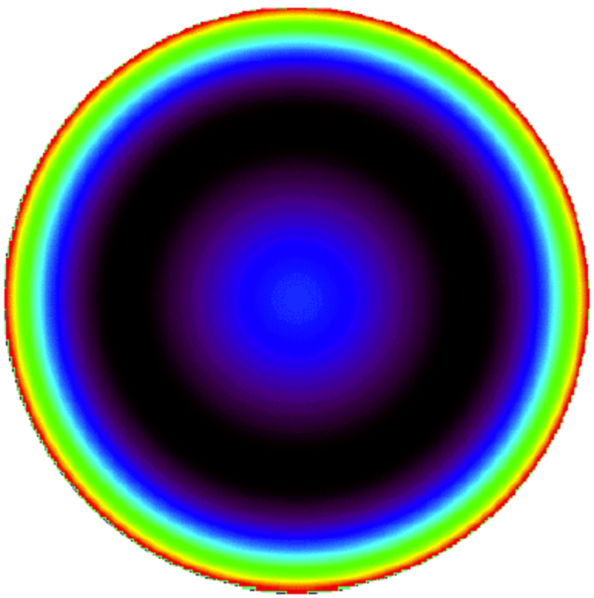
$Z_{2,-2}$



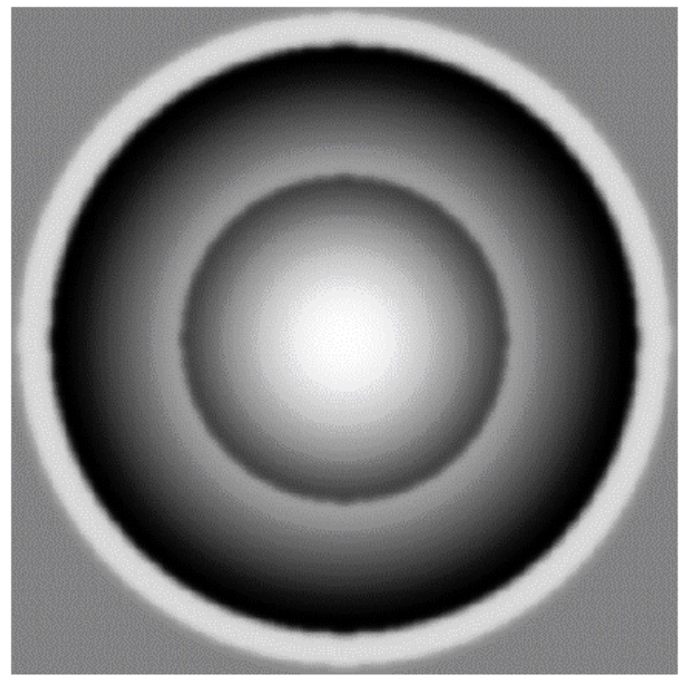
Difference Image

Credit: G. Chanan

Third order spherical aberration



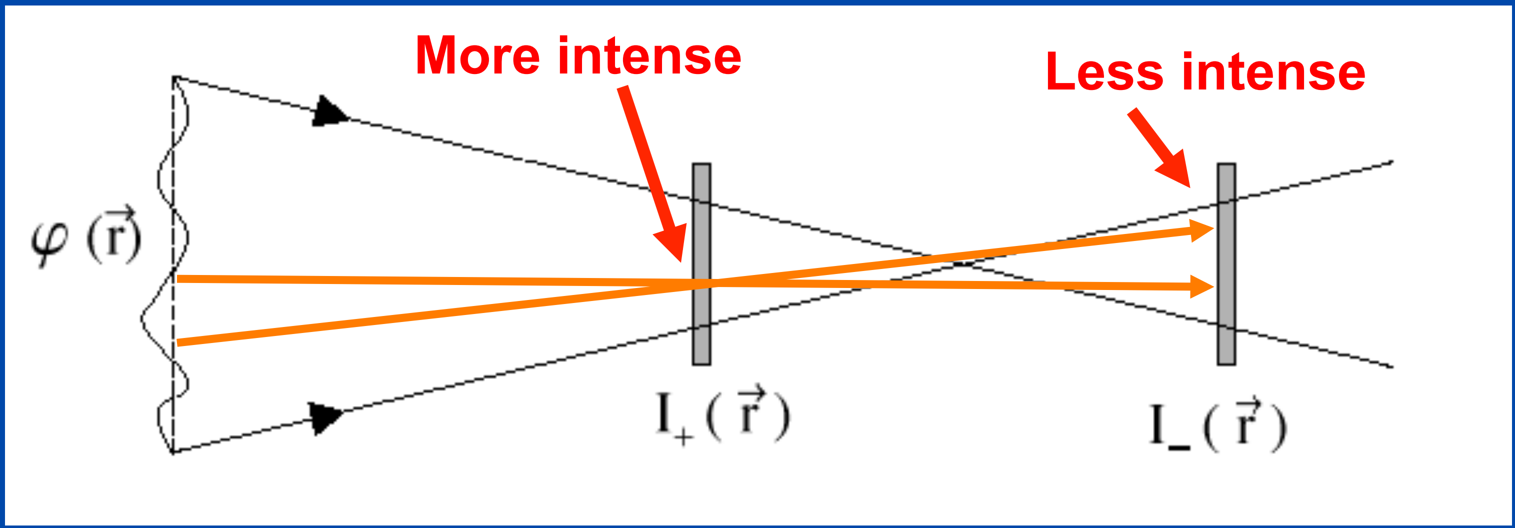
$Z_{4,0}$



Difference Image

Credit: G. Chanan

Practical implementation of curvature sensing



- Use oscillating membrane mirror (2 kHz!) to vibrate rapidly between I_+ and I_- extrafocal positions
- Measure intensity in each subaperture with an "avalanche photodiode" (only need one per subaperture!)
  - Detects individual photons, no read noise, QE ~ 60%
  - Can read out very fast with no noise penalty

Measurement error from curvature sensing

- Error of a single set of measurements is determined by photon statistics, since detector has NO read noise!

$$\sigma_{cs}^2 = \pi^2 \frac{1}{N_{ph}} \left(\frac{\theta_b d}{\lambda} \right)^2$$

where d = subaperture diameter and N_{ph} is no. of photoelectrons per subaperture per sample period

- Error propagation when the wavefront is reconstructed numerically using a computer scales poorly with no. of subapertures N:

$(\text{Error})_{\text{curvature}} \propto N$, whereas $(\text{Error})_{\text{Shack-Hartmann}} \propto \log N$

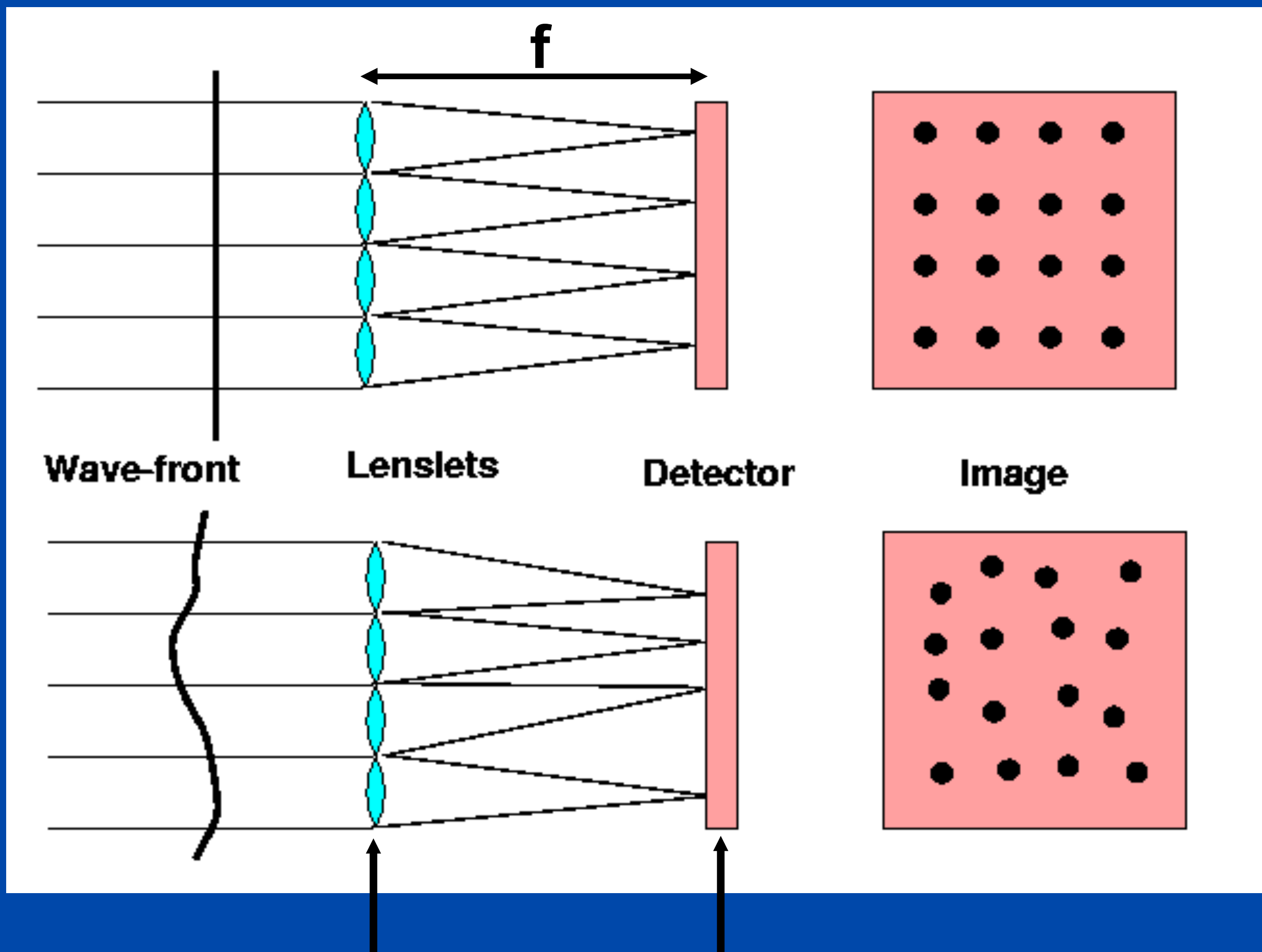
Question

- Think of as many pros and cons as you can for
  - Shack-Hartmann sensing
  - Curvature sensing

Advantages and disadvantages of curvature sensing

- Advantages:
  - Lower noise ⇒ can use fainter guide stars than S-H
  - Fast readout ⇒ can run AO system faster
  - Can adjust amplitude of membrane mirror excursion as "seeing" conditions change. Affects sensitivity.
  - Well matched to bimorph deformable mirror (both solve Laplace's equation), so less computation.
  - Curvature systems appear to be less expensive.
- Disadvantages:
  - Avalanche photodiodes can fail if too much light falls on them. They are bulky and expensive.
  - Hard to use a large number of avalanche photodiodes.
  - BUT – recently available in arrays

Review of Shack-Hartmann geometry



Pupil plane
Image plane

Pyramid sensing

- From Andrei Tokovinin’s tutorial

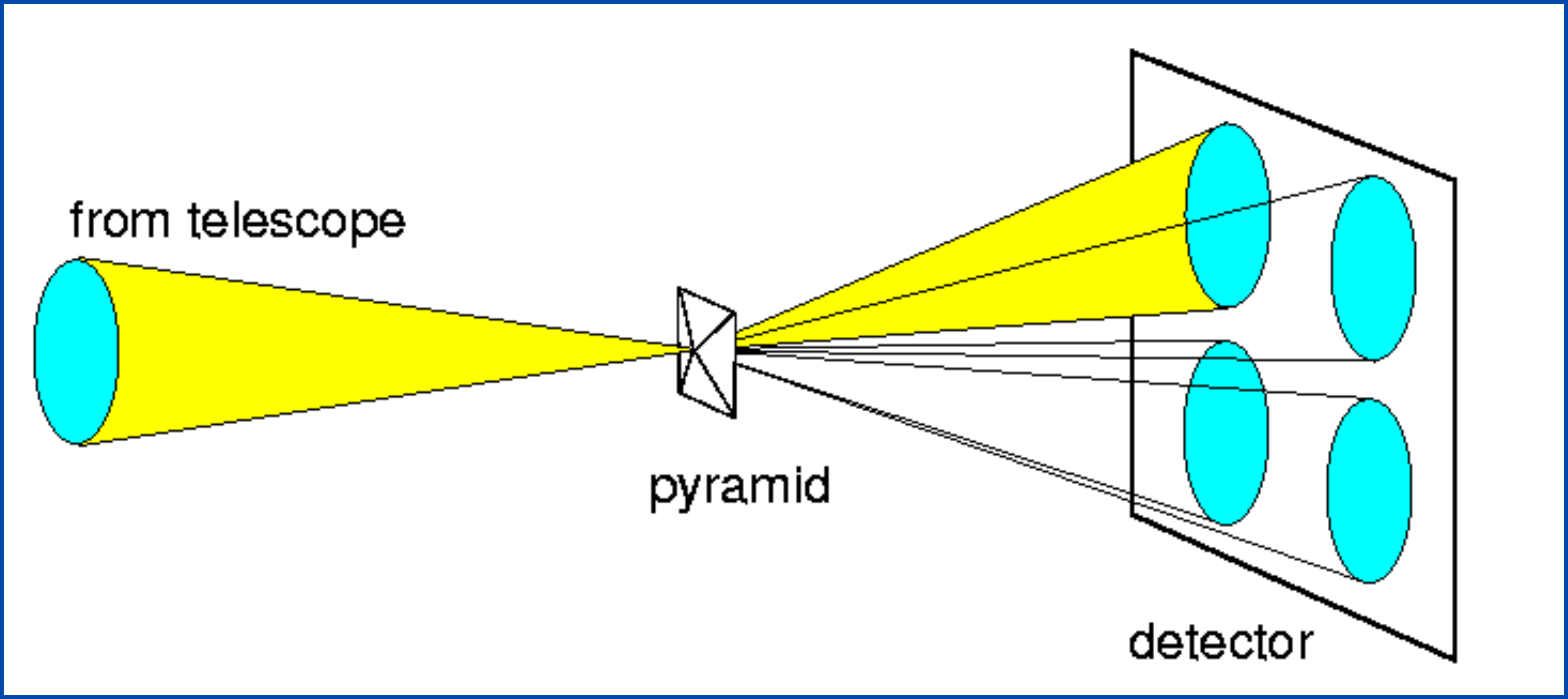
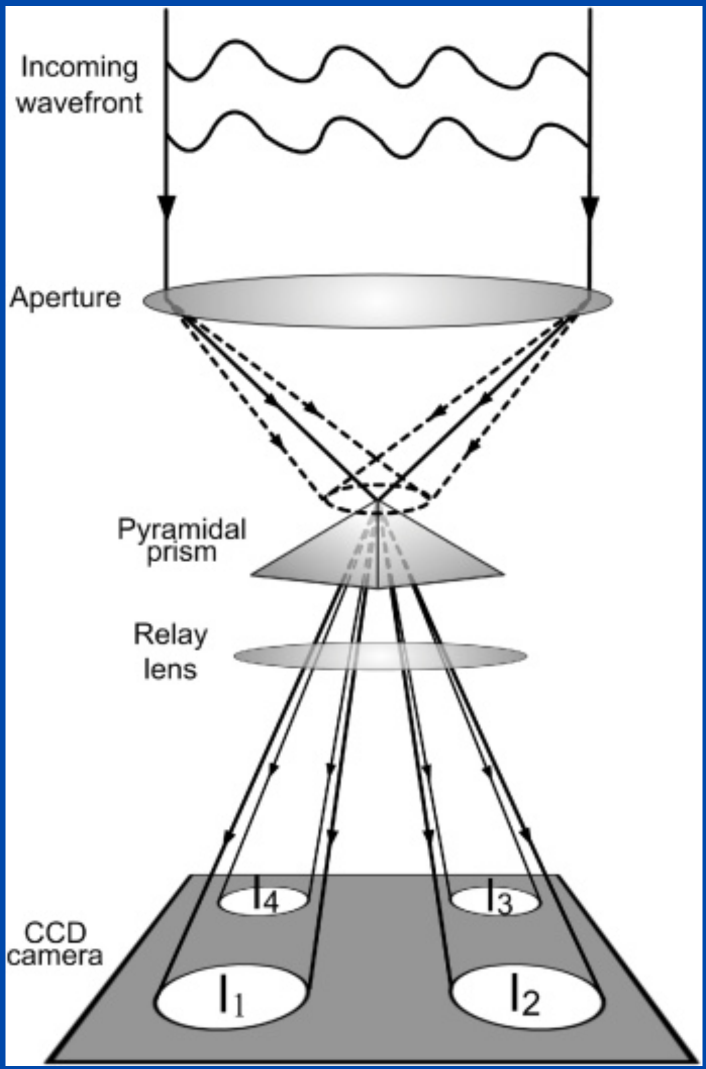


Image plane Pupil plane

Pyramid for the William Herschel Telescope's AO system

Schematic of pyramid sensor



Credit: Iuliia Shatokhina et al.

Pyramid sensor reverses order of operations in a Shack-Hartmann sensor

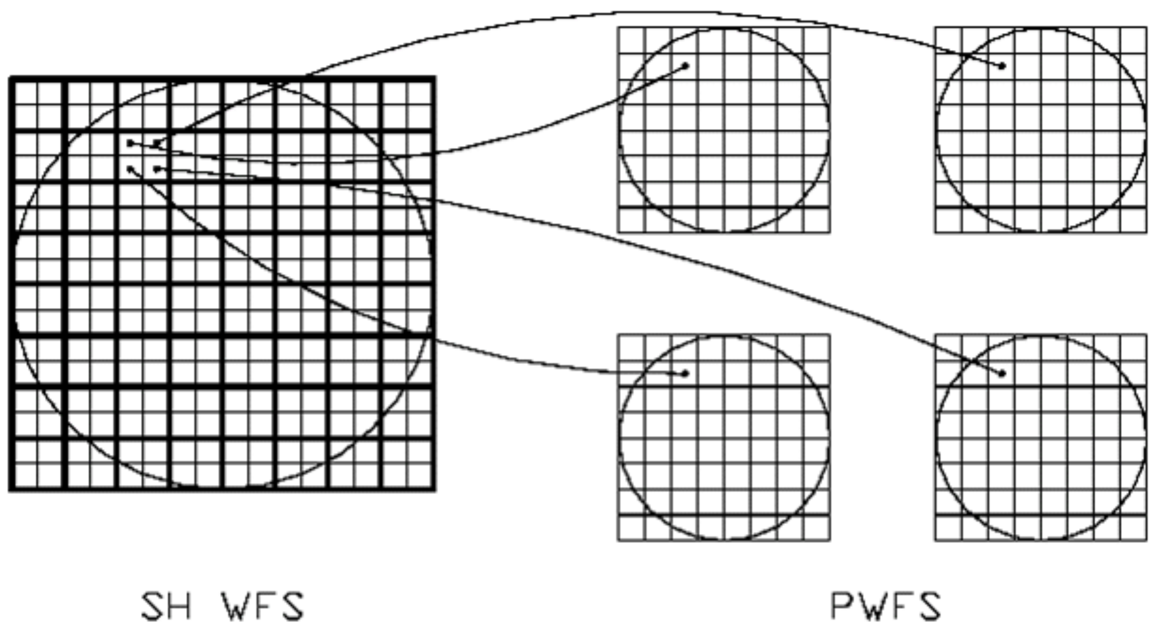
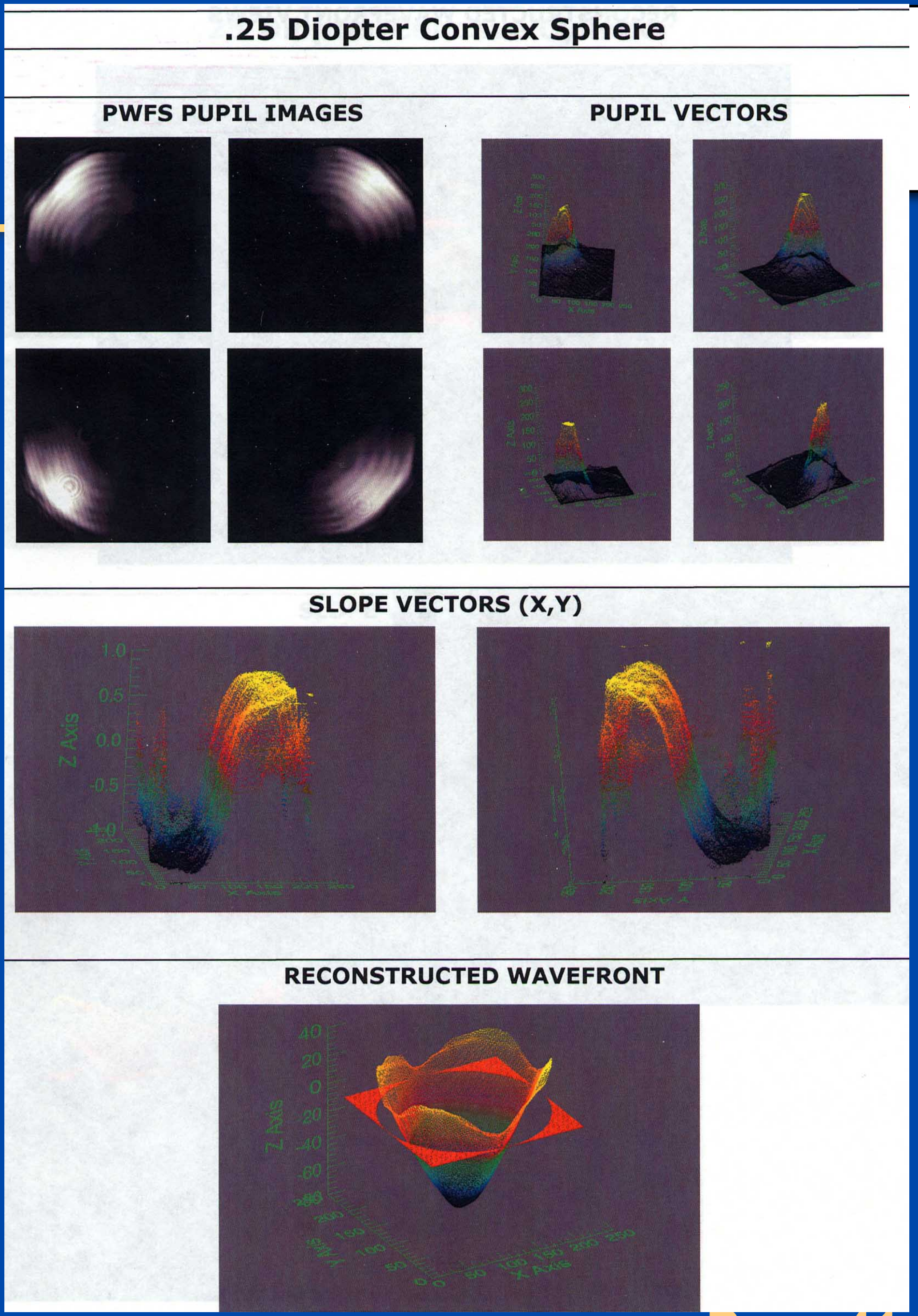
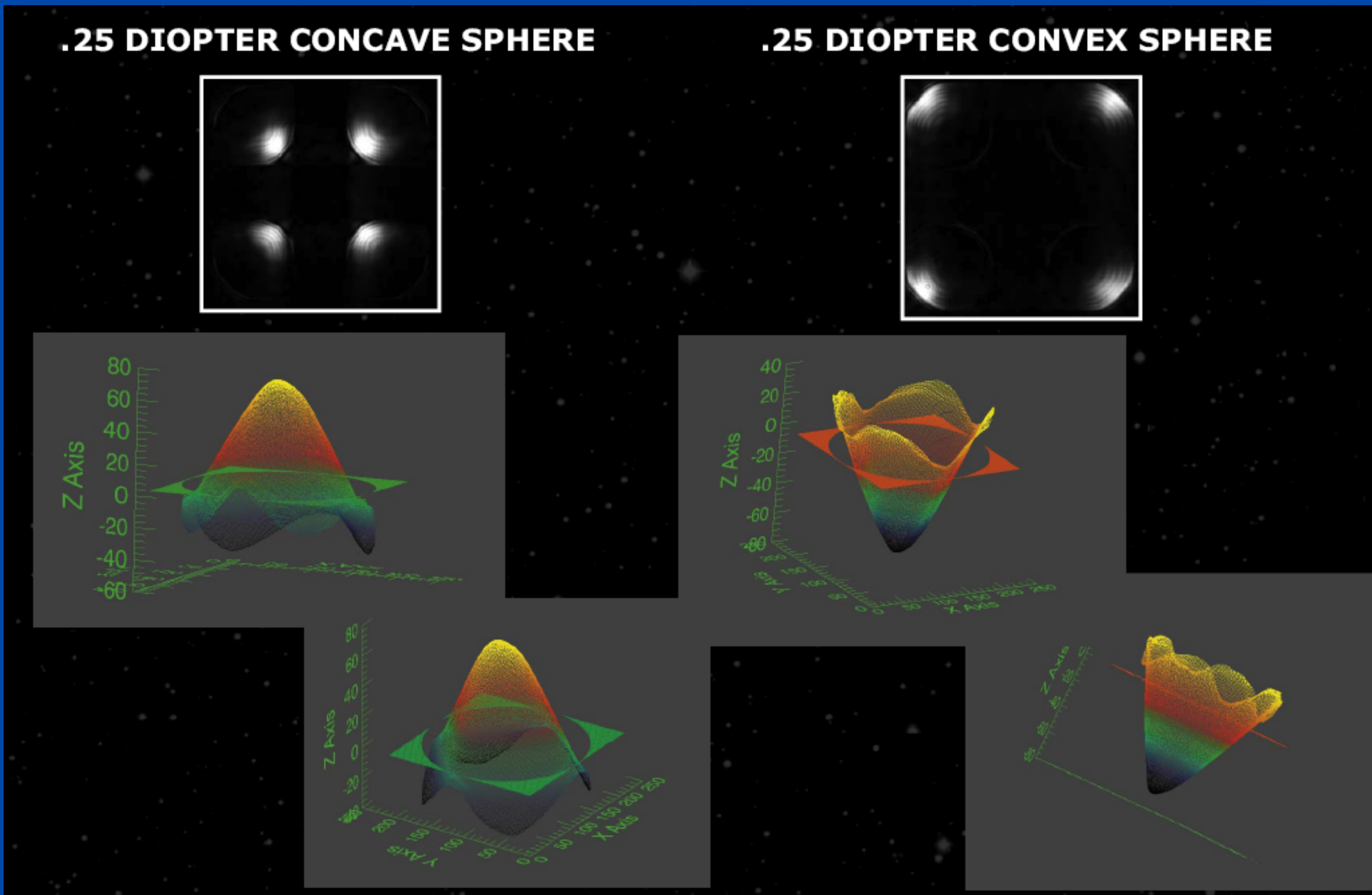


Figure 3- 4: Organization of SH wavefront data (left) versus pyramid wavefront data (right). The circle indicates the beam footprint on the WFS. The heavily-weighted squares on the left indicate the various subapertures (8x8 grid of subapertures). Each subaperture has 4 pixels (a quad cell). In a pyramid wavefront sensing scheme, each pixel represents a subaperture; the 4 images of the pupil correspond to the quadrants of the quad cell.

Here's what a pyramid-sensor meas't looks like

- Courtesy of Jess Johnson




.25 DIOPTER CONCAVE SPHERE
.25 DIOPTER CONVEX SPHERE
Z Axis
80
60
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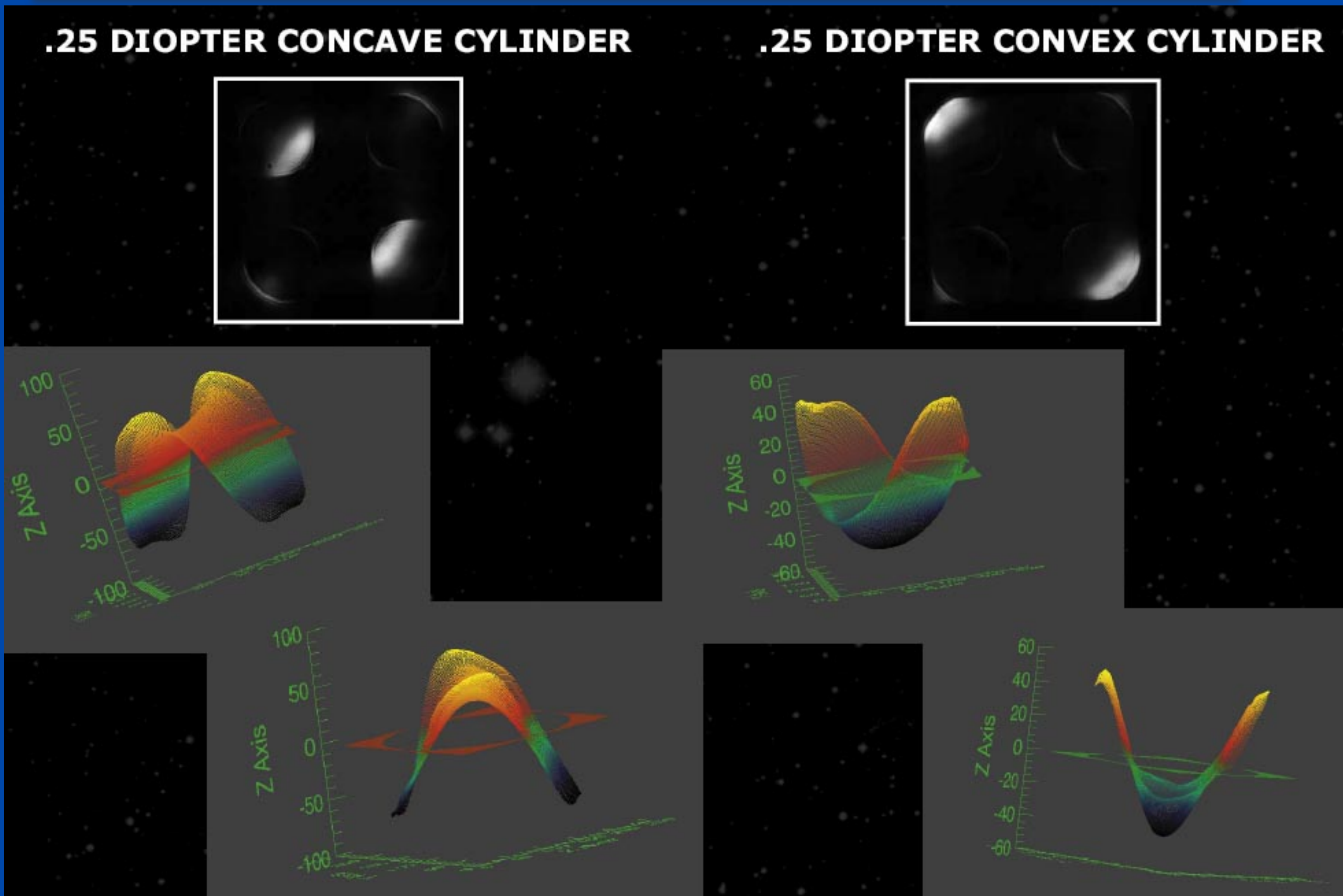
-60
X Axis
Y Axis


.25 DIOPTER CONCAVE CYLINDER
.25 DIOPTER CONVEX CYLINDER
Z Axis
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-40
-60

Potential advantages of pyramid wavefront sensors

- Wavefront measurement error can be much lower
  - Shack-Hartmann: size of spot limited to λ / d, where d is size of a sub-aperture and usually d ~ r_0
  - Pyramid: size of spot can be as small as λ / D, where D is size of whole telescope. So spot can be D/r_0 = 20 - 100 times smaller than for Shack-Hartmann
  - Measurement error (e.g. centroiding) is proportional to spot size/SNR. Smaller spot = lower error.
- Avoids bad effects of charge diffusion in CCD detectors
  - Fuzzes out edges of pixels. Pyramid doesn't mind as much as S-H.

Potential pyramid sensor advantages, continued

- Linear response over a larger dynamic range
- Naturally filters out high spatial frequency information that you can't correct anyway

Summary of main points

- Wavefront sensors in common use for astronomy measure intensity variations, deduce phase. Complementary.
  - Shack-Hartmann
  - Curvature sensors
- Curvature systems: cheaper, fewer degrees of freedom, scale more poorly to high no. of degrees of freedom, but can use fainter guide stars
- Shack-Hartmann systems excel at very large no. of degrees of freedom
- New kid on the block: pyramid sensors
  - Very successful for fainter natural guide stars